Other Facets of Giant Branch Evolution

• As the envelope cools due to expansion, the opacity in the envelope increases (due to Kramers law), so by the time the star reaches the base of the giant branch (point 5), convection dominates energy transport. The thermal energy trapped by this opacity causes the star to further expand. (Note: the expansion comes solely at the expense of thermal energy of the envelope.) Meanwhile, near the surface, H^- opacity dominates, and, as the star cools, the surface opacity becomes less. The energy blanketed by the atmosphere escapes, and the luminosity of the star further increases.

• As the star ascends the giant branch, the decrease in envelope temperature due to expansion guarantees that energy transport will be by convection. The convective envelope continues to grow, until it almost reaches down to the hydrogen burning shell. Recall that in higher mass stars, the core decreased in size during main-sequence evolution, leaving behind processed CNO. As a result, the surface abundance of ^{14}N grows at the expense of ^{12}C, as the processed material gets mixed onto the surface. **This is called the first dredge-up.** Typically, this dredge-up will change the surface CNO ratio from $1/2 : 1/6 : 1$ to $1/3 : 1/3 : 1$; this result is roughly independent of stellar mass.

• As the hydrogen-burning shell approaches the convective envelope, the latter retreats slightly, due to the radiation pressure, leaving behind a chemical discontinuity. When the hydrogen burning shell reaches this discontinuity, the luminosity of the giant star temporarily decreases, due to the abrupt changes in the mean molecular weight and the opacity. (This is easily demonstrated via shell homology.)

• For stars with $\mathcal{M} > 2.25\mathcal{M}_\odot$, the central core temperature continues to increase as the star ascends the giant branch. However,

in stars with $\mathcal{M} < 2.25\mathcal{M}_\odot$, electron degeneracy becomes important. This causes a drop in the opacity (due to heat conduction by electrons) and allows the core to radiate its energy more readily. At the same time, as the core compresses and the Fermi energy rises, some of the core's thermal energy must go into increasing the non-thermal kinetic energy of the electrons. Consequently, the core temperature drops (and, because of the decreased opacity, the core becomes even more isothermal).

• The drop in core temperature for low mass stars due to electron degeneracy is only temporary; after a while, the central temperature once again increases. However, because electron conduction is now important, the temperature rise is not a great as it would be in the ideal gas case. (The energy is transported away faster.) Low mass stars will rise high above the base of the giant branch before they ignite helium.

Helium Burning

Because there is no stable nucleus with $A = 5$ or $A = 8$, it is extremely difficult to get helium to fuse. In fact, there is no chain of light element reactions that can hurdle the $A = 8$ gap. Thus, to fuse helium, you must bring 3 nuclei together.

The key to fusing helium is the ^{8}Be nucleus. The ground state energy of the ^{8}Be nucleus is 91.78 keV higher than the ground state energy of two ^{4}He nuclei, hence ^{8}Be decays very quickly. However, because this energy difference is small (recall that most of the time we are working in MeV units), the decay is not instantaneous: the ^{8}Be nucleus has a lifetime of $\tau = 2.6 \times 10^{-16}$ sec. This is $\sim 10^5$ times longer than the duration of a normal scattering encounter, and sufficiently long so that some ^{8}Be will exist in the star. Specifically, the amount of ^{8}Be will be given by the equilibrium concentration of the reaction

$$^4\mathrm{He} + {}^4\mathrm{He} \rightleftarrows {}^8\mathrm{Be} \tag{22.1}$$

Thus, in principle, some triple-alpha fusion can occur.

Although it is possible compute the rate of fusion for helium using its quantum mechanical cross-sections, a simpler way is take advantage of the fact that ^{8}Be will have its equilibrium concentration. The physics which describes the equilibrium concentration of nuclear species is identical to that describing the concentration of atomic species. We can therefore calculate the abundance of ^{8}Be using the nuclear equivalent to the Saha equation. According to the Saha equation, the fractional abundance of ionized atoms is

$$\frac{n_{i+1}}{n_i} = \frac{u_e}{n_e} \frac{u_{i+1}(T)}{u_i(T)} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT} \tag{5.2.5}$$

where $u_e = 2$ is the statistical weight of the electron. The equation for the equilibrium abundance of ^{8}Be has the same form, with the "ionized" state being that which occurs when one ^{4}He nucleus is "ionized" from the other. Since ^{4}He and ^{8}Be are both in their ground state (with zero spin), their statistical weights are unity, so $u_i = u_{i+1} = u_e = 1$. Furthermore, recall that the $2\pi m_e kT$ term comes about from considering the momenta of free electrons relative to the atom. Here, the free particle is another ^{4}He nucleus, so clearly the appropriate mass to use is the reduced mass of the system.

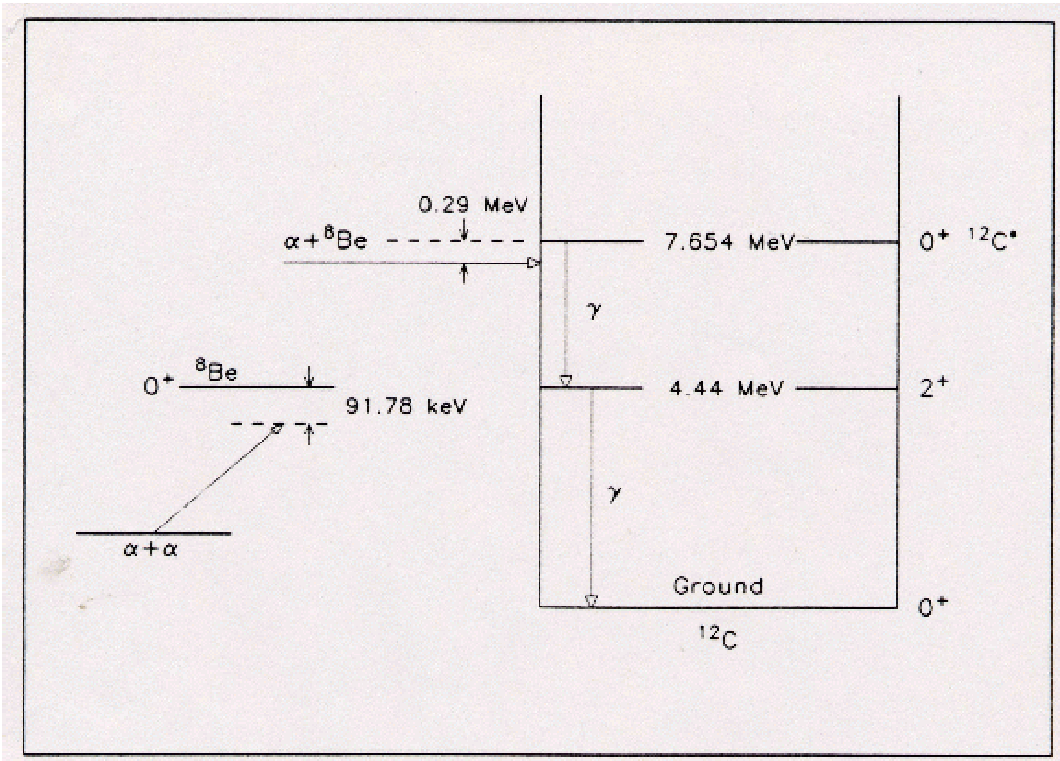
$$\mu_{\alpha\alpha} = \frac{m_{^4\mathrm{He}} m_{^4\mathrm{He}}}{m_{^8\mathrm{Be}}} \approx \frac{m^2_{^4\mathrm{He}}}{2m_{^4\mathrm{He}}} \approx \frac{m_{^4\mathrm{He}}}{2}$$

Thus the nuclear Saha equation for ^{8}Be is

$$\frac{N(^4\mathrm{He})^2}{N(^8\mathrm{Be})} = f_{\alpha\alpha} \left(\frac{2\pi \mu_{\alpha\alpha} kT}{h^2} \right)^{3/2} e^{-\chi_{\alpha\alpha}/kT} \qquad (22.2)$$

where $f_{\alpha\alpha}$ is the electron shielding factor. Note that the energy of the two helium nuclei is 91.78 keV smaller than that of the beryllium nucleus. Thus χ is negative, and the exponent of the Saha equation is positive. Plugging in the numbers for a typical core on the giant branch ($\rho \sim 10^5$ g cm^{-3} and $T \sim 10^8$ K) results in ~ 1 ^{8}Be nucleus for every 10^9 helium nuclei.

With the abundance of ^{8}Be now known, the next step in computing the triple-alpha reaction rate is to consider the reaction $^8\mathrm{Be}(\alpha, \gamma)^{12}\mathrm{C}$. This is a resonant reaction that goes to the excited state $^{12}\mathrm{C}^*$. (Actually, if it were non-resonant, there would never be enough ^{8}Be to make it proceed.) However, once in the $^{12}\mathrm{C}^*$ state, the particles encounter another problem.



There are three ways out of the 0^+ excited state of ^{12}C. The first, a direct decay to the ground state is highly forbidden (since the states have the same spin). The second, a decay to the ground state via the intermediate 2^+ state is also very unlikely. Consequently, the most likely scenario (by far) is for the ^{12}C* nucleus to spontaneous decay back into ^{8}Be + ^{4}He. Therefore, the computation of the triple-alpha reaction rate requires knowing both the amount of ^{12}C* present and the rate ^{12}C* decays to ground state ^{12}C.

The former computation is simple: the $^8\mathrm{Be}(\alpha,\gamma)^{12}\mathrm{C}$ reaction will happen quite rapidly (at least, when $T \gtrsim 10^8$ K); similarly, the inverse decay reaction will also be rapid. As a result, the two reactions will come to statistical equilibrium, and once again, the nuclear Saha equation can be applied. The amount of ^{12}C* will

thus be

$$\frac{N(^8\mathrm{Be})N(^4\mathrm{He})}{N(^{12}\mathrm{C}^*)} = f_{\alpha\mathrm{Be}}\left(\frac{2\pi\mu_{\alpha\mathrm{Be}}kT}{h^2}\right)^{3/2} e^{-\chi_{\alpha\mathrm{Be}}/kT} \tag{22.3}$$

where again, $f_{\alpha\mathrm{Be}}$ is the electron shielding factor, and $\mu_{\alpha\mathrm{Be}}$ is the reduced mass for the encounter.

Combining this with (22.2) we get

$$\frac{N(^4\mathrm{He})^3}{N(^{12}\mathrm{C}^*)} = f_{\alpha\alpha}\, f_{\alpha\mathrm{Be}}\, \mu_{\alpha\alpha}^{3/2}\, \mu_{\alpha\mathrm{Be}}^{3/2}\left(\frac{2\pi kT}{h^2}\right)^3 e^{-(\chi_{\alpha\alpha}+\chi_{\alpha\mathrm{Be}})/kT}$$

Since

$$\mu_{\alpha\alpha}\,\mu_{\alpha\mathrm{Be}} = \left(\frac{m_{^4\mathrm{He}}^2}{m_{^8\mathrm{Be}}}\right)\left(\frac{m_{^4\mathrm{He}}m_{^8\mathrm{Be}}}{m_{^{12}\mathrm{C}}}\right) \approx \frac{m_{^4\mathrm{He}}^3}{3m_{^4\mathrm{He}}} \approx \frac{m_{^4\mathrm{He}}^2}{3}$$

we have

$$N(^{12}\mathrm{C}^*) = N(^4\mathrm{He})^3 f_{\alpha\alpha}\, f_{\alpha\mathrm{Be}}\frac{3^{3/2}h^6}{(2\pi m_{^4\mathrm{He}}kT)^3}\, e^{-\chi/kT} \tag{22.4}$$

where $\chi = 379$ keV is the net energy difference between three ^{4}He nuclei and ^{12}C*.

Finally, to compute the rate of ^{12}C* decays to ground state, we can use uncertainty principle, which says that the decay rate of a transition, λ, is related to its energy width, Γ by

$$\Gamma\,\tau = \frac{\Gamma}{\lambda} \sim \hbar \tag{22.5}$$

For the $0^+\!\rightarrow\!2^+$ transition of ^{12}C, $\Gamma \sim 2.8\times10^{-3}$ eV, which means that a single ^{12}C* would have to wait over 100,000 years for this

decay! Nevertheless, at any instant, there are a lot of $^{12}\mathrm{C}^*$ nuclei; the reaction rate per unit volume is given by $N(^{12}\mathrm{C}^*)\,\Gamma/\hbar$, and the reaction rate per unit mass is $N(^{12}\mathrm{C}^*)\,\Gamma/\hbar\rho$, or

$$r_{3\alpha} = \frac{Y^3\rho^2 N_A^3}{A^3}\, f_{\alpha\alpha}\, f_{\alpha\mathrm{Be}} \left(\frac{\Gamma}{\hbar}\right) \frac{3^{3/2}h^6}{(2\pi m_{^4\mathrm{He}}kT)^3}\, e^{-\chi/kT} \tag{22.6}$$

Since three ^{4}He are destroyed for every ^{12}C fusion, the lifetime of ^{4}He against the triple-alpha process is

$$\tau_{3\alpha} = \frac{N(^4\mathrm{He})}{3r_{3\alpha}} \tag{22.7}$$

and if the energy per reaction is $Q = 7.275$ MeV, then the net energy per gram produced by the triple-alpha process is

$$\epsilon_{3\alpha} = r_{3\alpha}Q = 5.08\times 10^8\ f_{\alpha\alpha}\, f_{\alpha\mathrm{Be}} \frac{\rho^2 Y^3}{T_9^3} e^{-4.4027/T_9}\ \mathrm{ergs\ g^{-1}\ s^{-1}} \tag{22.8}$$

Note that the reaction depends on the square of the density: this makes sense since it is essentially a three-body reaction. Note also that the temperature coefficient is

$$\nu = \frac{d\ln\epsilon}{d\ln T} = -3 + \frac{4.40}{T_9} \tag{22.9}$$

which, for a characteristic core temperature of $T \sim 10^8$ K implies $\epsilon_{3\alpha} \propto T^{40}$. Thus, helium burning can be explosive!

(Note that the above analysis is only correct for temperatures above $\sim 10^8$ K. At lower temperatures, the ^{8}Be and ^{12}C reactions are sufficiently slow so that equilibrium may not be achieved. Moreover, at lower temperatures, electron screening begins to move from the

weak screening to the strong screening limit. This makes the calculation much tougher.)

Once ^{12}C is formed, another ^{4}He capture can take place to make ^{16}O, while releasing $Q = 7.162$ MeV. Unfortunately, the rate for this reaction is extremely uncertain. It is presumed to proceed through the high-energy tail of a nuclear resonance whose properties are not well known. The rate is thus uncertain by at least a factor of two. A best guess at the energy generation rate is

$$\epsilon_{\alpha^{12}\mathrm{C}} = 1.5 \times 10^{25} \, \frac{\rho Y X_{^{12}\mathrm{C}}}{T_9^2} \left(1 + 0.0489 T_9^{-2/3}\right)^{-2}$$
$$\exp\left\{-32.12 T_9^{-1/3} - (0.286 T_9)^2\right\} \text{ ergs g}^{-1} \text{ s}^{-1} \tag{22.10}$$

Clearly, the rate at which ^{16}O forms will increase with time, as the abundance of ^{12}C increases.

If ^{16}O is made, then it is possible to make ^{20}Ne as well, via fusion with another ^{4}He nucleus. At the nominal core temperature of 10^8 K, this reaction takes place through the extreme tails of two different nuclear resonances which bracket the Gamow peak. As a result, the non-resonant reaction formula is usually used. Because the Coulomb repulsion is significant for ^{16}O, and because $^{16}\mathrm{O}(^4\mathrm{He}, \gamma)^{20}\mathrm{Ne}$ requires a fair amount of ^{16}O to already exist, this reaction takes place at a lower rate than the other two reactions. However, as the core temperature increases, the ^{20}Ne resonance at the high end of the Gamow peak becomes more important, and the rate will increase. (The high energy resonance is not that important for red giant branch burning, but it can be important for later stages of stellar evolution.) The Q-value for this reaction is

$Q = 4.734$ MeV; the reaction rate is

$$\epsilon_{\alpha^{16}\mathrm{O}} = 6.69 \times 10^{26} \, \frac{\rho Y X_{^{16}\mathrm{O}}}{T_9^{2/3}} \exp\left\{-39.75 T_9^{-1/3} - (0.631 T_9)^2\right\} \text{ ergs g}^{-1}\text{ s}^{-1} \tag{22.11}$$